

Super-Planckian thermal emission from a hyperlens

C. Simovski,^{1,2} S. Maslovski,^{3,2} S. Tretyakov,¹ I. Nefedov,¹ S. Kosulnikov,² and P. Belov²

¹*Department of Radio Science and Engineering, Aalto University, P.O. Box 13000, FI-00076, Aalto, Finland*

²*Laboratory of Metamaterials, University for Information Technology, Mechanics and Optics (ITMO), St. Petersburg 197101, Russia*

³*Departamento de Engenharia Electrotécnica, Instituto de Telecomunicações, Universidade de Coimbra, Pólo II, 3030-290 Coimbra, Portugal*

We suggest and theoretically explore a possibility to strongly enhance the steady thermal radiation of a small thermal emitter using an infrared hyperlens. The hyperbolic metamaterial of the hyperlens converts emitter's near fields into the propagating waves which are efficiently irradiated from the hyperlens surface. Thus, with the hyperlens, emitter's spectral radiance goes well beyond the black-body limit for the same emitter in free space. Although the hyperlens can be kept at a much lower temperature than the emitter, the whole structure may radiate, in principle, as efficiently as a black body with the same size as that of the hyperlens and the same temperature as that of the emitter. We believe that this study can lead to a breakthrough in radiative cooling at microscale, which is crucial for microlasers and microthermophotovoltaic systems.

PACS numbers: 44.40.+a, 42.25.Bs, 78.20.nd

In the classical theory of thermal radiation the power radiated from a unit surface of an optically large body in free space per unit interval of frequencies is given by Planck's formula:

$$P_{\omega}^{\text{FS}} = \pi e_s(\omega) B_{\omega} = \frac{\hbar \omega^3 e_s(\omega)}{4\pi^2 c^2} \left[e^{\hbar \omega / (k_B T)} - 1 \right]^{-1}, \quad (1)$$

where k_B and \hbar are Boltzmann's and Planck's constants, respectively, T is the temperature of the body surface and $e_s(\omega) < 1$ is the spectral emissivity of body's material. For a black body (BB) emitting, in accordance to the Planckian theory, maximal thermal radiation to free space, $e_s(\omega) \equiv 1$. For a material having no optical losses at frequency ω , i.e. for transparent media, $e_s(\omega) = 0$ and the emission is absent. It is commonly accepted that the thermal radiation is non-coherent and its spatial distribution is isotropic. Both these factors result in Lambertian pattern for a radiating half-space. However, recent investigations have shown that thermal radiation can be partially coherent [1], directive [2], and combining coherence and directionality [3, 4]. These deviations originate from intrinsic properties of metamaterials [5]. Especially, the so-called hyperbolic metamaterial (HMM) shows interesting responses to thermal radiation (see e.g. in [6–9]). In this paper we theoretically reveal a possibility for a sample of HMM to strongly enhance the far-field radiation from small (several micrometers) emitters exceeding the BB limit defined for the emitters of the same size in free space. To our knowledge, in previous works related with applications of HMMs for radiative heat transfer these materials were used only to control near-field thermal flows. Here, we use these media to enhance far-field radiation.

Since classical works by Kirchhoff and Planck, the BB has been considered as a perfect thermal emitter whose spectral radiance cannot be exceeded in far-field zone (so-called Planckian limit). Despite that the photonic density of states (PDOS) and, consequently, the rate of spontaneous emission responsible for the thermal radiation may be enhanced considerably (e.g. in [10] by one order of magnitude) there is a belief that the photons that occupy the extra available states cannot be emitted out of a medium with high PDOS [11] due to the total internal reflection (TIR). However, it is not generally true.

Long ago, in work [12], a possibility to exceed the BB limit for a hot particle having resonant sizes at infrared was pointed out. Recently, the authors of [13] have demonstrated super-Planckian radiation from a macroscopic emitter achieved due to a transparent dielectric dome. For a hemispherical emitter this idea is illustrated in Fig. 1 (left half). As it follows from Eq. (1), filling free space with an isotropic transparent medium with refractive index $n = \sqrt{\epsilon_h}$ [respectively, c is replaced by c/n in Eq. (1)] increases the radiated power by n^2 , provided that $e_s(\omega)$ stays unchanged. If such transparent medium forms a lens in a shape of a hemispherical dome as is shown in Fig. 1, the emitted waves impinge on the lens surface and, after being partially reflected, pass onto free space. When the dome radius R is much larger than the emitter radius r , the power transmittance to the free space for these waves can be found in geometric optics (GO) approximation as for normally incident rays: $t_{\text{GO}} = 4n/(n+1)^2$. Thus, the total gain in the power irradiated to the far zone due to the presence of the dome equals $G = n^2 \times t_{\text{GO}} = 4n^3/(n+1)^2$. From here it may seem that one can achieve arbitrary high gain when $n \rightarrow \infty$. However, this is not true, because when $n \gtrsim R/r$ some of the incident rays start experiencing TIR at the output interface of the dome. This effect also ensures that the apparent diameter of the emitter as is seen from outside of the dome never exceeds the diameter of the dome, which sets an obvious upper bound for the total gain in this structure when $R \gg \lambda$: $G < R^2/r^2$, i.e., the whole structure may not radiate more than a BB with radius equal to the outer radius of the dome.

Because realistic thermal sources have $e_s < 1$, and because the known transparent materials in the infrared range have rather small refractive indices $n \lesssim 3$, the thermal lens [13] can hardly offer gain G_{BB} which would exceed 7. Here, G_{BB} is the ratio of power emitted by a realistic thermal source covered with a transparent dome to the power emitted

by an uncovered BB with the same size and temperature as the original source. In [13] the gain $G_{\text{BB}} \approx 3.1$ has been experimentally demonstrated for a thermal lens of centimeter size with $n \approx 2.4$. The above estimations predict $G_{\text{BB}} \approx 4.8$ for this case, when the emitter is an ideal BB.

The study that we are going to present next has been motivated by the following question: Since it is possible to enhance the thermal radiation of an emitter by 2–5 times by using a hemisphere of a transparent isotropic dielectric, can we go further using more advanced materials? Namely, can we approach the GO bound: $G_{\text{max}} = R^2/r^2$ with these materials? Note that here we are interested in the case when $R \approx 3\lambda$ or greater, because bodies with $R \lesssim \lambda$ can outperform this bound [12]. We show that a dome made of a hyperbolic metamaterial theoretically allows one to increase the spectral radiance of small emitters by up to two orders of magnitude, as compared to the limit dictated by Planck's law for BB emitters of the same size in vacuum. Hyperbolic metamaterials (HMM) which we propose for this purpose are uniaxial dielectric composites with the permittivity tensor defined by two components: transverse ε_{\perp} and axial ε_{\parallel} , such that $\text{Re}(\varepsilon_{\parallel})\text{Re}(\varepsilon_{\perp}) < 0$, $\text{Re}(\varepsilon_{\parallel,\perp}) \gg \text{Im}(\varepsilon_{\parallel,\perp})$. The isofrequency surfaces (also called wave-dispersion surfaces) for HMM represent hyperboloids. A hot unit volume inside a HMM sample emits much more electromagnetic energy than a unit volume of a conventional lossy medium at the same temperature. This effect results from high Purcell's factor of a dipole located inside HMM. The concept of Purcell's factor (the gain in the spontaneous emission rate) historically referred to the case when the dipole radiation was enhanced by a closely located resonator (see e.g. in [14]). However, in work [15] the notion of Purcell's factor was extended to any environment of the dipole source different from free space. Purcell's factor of HMM dramatically exceeds the Purcell's factor $F_{\text{P,die}} = n$ of a usual dielectric. In the lossless HMM without internal granularity the radiation resistance of a point dipole oriented orthogonally to the optical axis tends to infinity [16, 17] because all the power is irradiated in the form of propagating waves. Therefore the thermal radiation of a unit hot volume in such an ideal HMM should be infinite. For realistic (lossy and internally granular) HMM the thermal radiation of a unit hot volume is finite but strongly super-Planckian as compared to vacuum [6, 7].

The excessive super-Planckian radiative heat in HMM is contained in the modes with high transverse wavenumbers $q = 2\pi/\Lambda_{\text{tr}} \geq \omega/c$. In flat uniaxial HMM slabs with the optical axis oriented orthogonally to the interface plane these modes experience TIR at the interface with free space and, thus, are confined inside the HMM (note that the coupling of such modes with free space can be carried out in asymmetric HMM [18–20], where the optical axis is tilted to the slab interface). As a result, the thermal radiation from such slabs into free space does not exceed the BB limit. However, it can be very close to it, and it is known that a half-space of HMM mimics the BB [5–7].

On the other hand, in locally uniaxial radially symmetric HMM samples the eigenmodes which are characterized with high local wavenumbers $q(r) \gg \omega/c$ close to the center of the sample, may attain $q(R) < \omega/c$ at enough large radial distance $R \gg r$, because in these modes, roughly, $q(r) \propto 1/r$. Therefore, in radially symmetric HMM these modes can couple to the free space propagating waves if the radius R is large enough. This effect is known as hyperlensing [21–23]. Hyperlenses (HLs) were previously designed for obtaining magnified images of subwavelength objects.

In fact, HL is also a matching device for the radiation propagating from its central part to free space [24]. Here, we suggest to use a dome of radially symmetric HMM which operates as an infrared HL to extract the excessive super-Planckian heat otherwise confined within emitter's near field in the modes with high transverse wavenumbers. An implementation of such HMM in the infrared range is, for example, an optically dense array of aligned metal nanowires called wire medium (WM). WM is a spatially dispersive implementation of HMM [25]. The spatial dispersion of HMM for our purpose is not a harmful factor. On the contrary, in accordance to our estimations the spatial dispersion helps to match the hyperlens to free space.

Performing our emitter as a lossy dielectric body placed inside a transparent dielectric dome both comprising radially divergent nanowires, we arrive at the structure sketched in Fig. 1 (right half). For better matching of the HL to free space

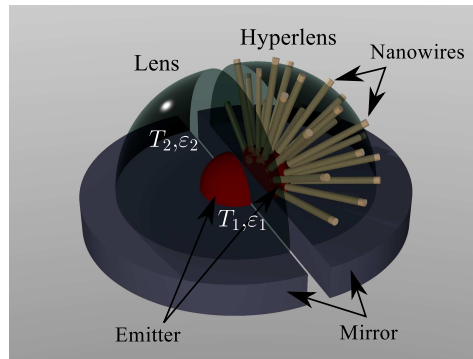


FIG. 1: Two hemispherical structures which offer super-Planckian thermal radiation from a finite emitter (just a half of each structure is shown). Left: Thermal lens analogous to the one considered in Ref. [13]. Right: Thermal hyperlens (HL) comprising radially diverging nanowires. The lens and HL's host are made of transparent glass with permittivity $\varepsilon_2 \equiv \varepsilon_{\text{h}}$. The emitter is formed by a lossy medium with complex permittivity ε_1 and is partially filled with nanowires. The emitter is set under high temperature $T_1 \gg T_2$, where T_2 is the ambient temperature.

the ends of the nanowires can be made free-standing as it is shown in the figure. To prevent direct thermal contact, the nanowires in the emitter may be separated from the HL by a sufficiently narrow nanogap. It is critical that the nanowires are radially oriented in the whole structure and that their density decreases with radial coordinate. The divergence angle ϕ between adjacent nanowires should be small enough so that the properties of HMM in the emitter volume are preserved, however, large enough so that the best possible matching to free space is achieved.

Formulas for the effective permittivity of WM operating at infrared can be found in [25, 26]. We use highly radially anisotropic HMM, in which $\text{Re}(\varepsilon_{\perp}) > 0$, $\text{Re}(\varepsilon_{\parallel}) < 0$, $|\varepsilon_{\parallel}| \gg |\varepsilon_{\perp}|$, and the energy propagates roughly in the radial direction, independently on the value of q . Thus, we notice that in this regard the situation is similar to the case of the simple dielectric dome considered previously, with a difference that when estimating the power transmittance through the outer interface of the HL we must use the effective complex index of refraction in the WM in the vicinity of the outer interface: $n_{\perp,\text{out}} = \sqrt{\varepsilon_{\perp,\text{out}}}$. Hence, $t_{\text{HL}} = 4\text{Re}(n_{\perp,\text{out}})/|n_{\perp,\text{out}} + 1|^2$.

Note that still the modes with $q(R) \geq \omega/c$ experience TIR at the dome-air interface and for these modes $t_{\text{HL}} = 0$. From recent studies of the dipole radiation in WM [27, 28] it is known that the irradiated wave beam is nearly as narrow as the WM period a . Therefore, in order to obtain high transmittance to free space (nearly as high as t_{HL}) for the dominant part of the spatial spectrum exited within HL, the divergence angle ϕ should be such that the separation between the nanowires at the outer surface of the HL is about $\lambda/2$ or larger. Hence, $\phi \gtrsim \lambda/(2R)$.

Let us now estimate how large can be the gain G_{HL} in the HL configuration of Fig. 1. First, we note that inclusion of nanowires into a dielectric host increases the power radiated by an elementary dipole placed inside this medium by F_{P} times, where F_{P} is the Purcell factor for uniaxial WM. This factor was calculated in Ref. [17]. Being averaged over all possible locations of a transversely oriented electric dipole, F_{P} equals

$$F_{\text{P}}^{\text{tr}} \approx \frac{3k_{\text{p}}^2}{8k_{\text{h}}^2} \log \left[1 + \frac{K_m^2}{k_{\text{p}}^2} \right], \quad (2)$$

where $k_{\text{h}} = \sqrt{\varepsilon_{\text{h}}}\omega/c$ and $k_{\text{p}} = \sqrt{2\pi/\log[a^2/4r_0(a-r_0)]}/a$ are the wavenumber in the host medium and the plasma wavenumber in WM [29], respectively, where a is the WM period and r_0 is the wire radius. In Eq. (2), K_m is the spatial spectrum cut-off parameter, which equals $2\sqrt{\pi}/a$ in unbounded uniaxial WM [17]. Because only the modes with $q < \omega/c$ are irradiated from HL's outer surface, here we must limit this parameter by $K_m = \min[2\sqrt{\pi}/a, (\omega/c)(R/r)]$. Strictly speaking, Eq. (2) refers to the case when the wires are perfectly conducting, however in [17] the estimations were done also for lossy wires and it was shown that (2) was applicable to realistic metal nanowires if they were thick enough (practically, their radius r_0 should be larger than the skin depth). For dipoles parallel to the wires the Purcell factor is much smaller than F_{P}^{tr} and can be neglected. Since a hot elementary volume of a lossy medium surrounded by nanowires can be treated as a set of three identical mutually orthogonal dipoles emitting thermal radiation, in thermal emission calculations we must use the average $F_{\text{P}}^{\text{HL}} = 2F_{\text{P}}^{\text{tr}}/3$, where F_{P}^{tr} is given by (2).

Under these conditions, the total gain due to the effect of the HL dome can be estimated as follows:

$$G_{\text{HL}} \approx F_{\text{P}}^{\text{HL}} \times \text{Re}(n_{\perp,\text{in}})^2 \times t_{\text{HL}} \times e^{-2\alpha(R-r)} \approx \frac{8F_{\text{P}}^{\text{tr}} \times \text{Re}(n_{\perp,\text{in}})^2 \text{Re}(n_{\perp,\text{out}})}{3|n_{\perp,\text{out}} + 1|^2} e^{-2\alpha(R-r)}, \quad (3)$$

where $n_{\perp,\text{in}}$ is the effective refractive index of the HL in the vicinity of the emitter (at the inner interface of the HL), and $\alpha \approx (\omega/c)\text{Im}(\sqrt{\varepsilon_{\perp}})$ is the decay factor due to the loss in the nanowires. Note that the structure suggested in this paper cannot radiate more than a BB with the same size as that of the dome and the same temperature as that of the emitter when $R \gg \lambda$. Moreover, because the WM-based HL interacts mostly with P -polarized waves, the actual upper bound for the gain in this case is $G_{\text{HL}}^{\text{max}} \sim 0.5R^2/r^2$.

Due to optical losses in the metal the decay of thermal radiation over the path $R \gg \lambda$ is not negligible. This factor restricts the radius R by dozens of λ . However, Eq. (3) does not take into account the thermal radiation of heated wires inside the hyperlens. This additional emission can significantly increase G_{HL} compared to (3), so that it may approach the GO bound: $G_{\text{max}} = R^2/r^2$. In the same time, (2) slightly overestimates the Purcell factor for realistic nanowires. So, the implementation of our thermal HL with macroscopic dimensions (e.g. with $R = 1$ cm like in [13]) is disputable. In the present study we deal with a microscopic hyperlens with radius $R = 10 \mu\text{m}$ and an emitter of radius $r = 0.75 \mu\text{m}$.

Estimations of the factor G_{HL} were done using Eq. (3) and the effective-medium model of infrared WM [26]. The material parameters of gold were taken from Ref. [30]. We considered a hemispheric structure with concentric hemispheric emitter located on a perfect mirror as in Fig. 1. The HL is performed of non-tapered gold nanowires with the thickness $2r_0 = 50$ nm located in a matrix with $\varepsilon_2 = 3.16$ (chalcogenide glass transparent in the range 50–150 THz). We calculate the gain G_{HL} in the range 100–140 THz, where $r_0 > \delta$ (δ is the skin-depth of gold), and formula (2) for Purcell's factor is applicable. Internal ends of nanowires are located at $r_1 = 0.5 \mu\text{m}$ from the geometrical center of the structure. An emitter comprises the hemisphere $r = 0.75 \mu\text{m}$ and is partially filled with nanowires. The emitter is assumed to be a lossy dielectric which is well impedance-matched with the HL. The distance between the centers of nanowires at the surface $r_1 = 0.5 \mu\text{m}$ equals 100 nm and within the emitter the averaged period of the WM equals $a = 125$ nm. Nanowires diverge with the angle $\phi \approx 10^\circ$. This angle is small enough to neglect the divergence of nanowires when calculating the effective permittivity of HMM in the domain of the emitter and its Purcell factor F_{P}^{tr} . However, it is large enough to offer good

matching of the HL to free space, because for $\phi \gtrsim 7.5^\circ$ the distance A between the axes of nanowires at the outer surface of the HL exceeds $\lambda/2$ at frequencies 100–140 THz.

Following to (2) Purcell's factor of the medium of parallel nanowires with the period $a = 125$ nm for a transverse electric dipole decreases from $F_P^{\text{tr}} \approx 18$ to $F_P^{\text{tr}} \approx 7.5$ over the range 100–140 THz. Then the relative enhancement G_{HL} of the power spectrum radiated by an arbitrary dipole \mathbf{p} located in between the wires near the internal surface of the HL in accordance to (3) is within approximately $40 \dots 20$ over this frequency range. The range 100–140 THz is around the maximum of emission for the emitter temperatures T_1 of the order 700–800°C. Higher temperatures are hardly actual for our HL since nanowires can melt. For lower temperatures thermal radiation is concentrated in lower frequencies where Purcell's factor is higher. For example, at 50 THz in unbounded WM $F_P^{\text{tr}} \approx 70$. The same divergence angle at this frequency implies larger R needed for matching the HL to free space. The condition $A > \lambda/2$ holds at 50 THz for $R = 20 \mu\text{m}$. Then, taking into account the decay we obtain using (3) the gain $G_{\text{HL}} \approx 170$ at 50 THz. So, for emitters with temperatures $T_1 < 500\text{--}600^\circ\text{C}$ the thermal radiation of the emitter within HL may exceed the BB limit for the same emitter in vacuum by two orders of magnitude.

To check our estimations of the gain G_{HL} we performed extensive numerical simulations. We studied a HL excited by a transverse dipole located in the middle between the ends of adjacent nanowires either at the surface of the central nanocavity or displaced from this surface – either embedded into the WM up (to 250 nm from the central cavity) or located inside it. The parameters of the HL in these simulations are as above besides one replacement – we substituted gold nanowires by perfectly conducting ones. This replacement dramatically reduced the computation time needed for the structure comprising many hundreds of metal nanowires and made simulations realistic. Simulations were performed using the CST Studio Suit software.

Although replacing gold by perfect conductor we removed the decay factor $\exp(-2\alpha R)$, this is still a reasonable model of a HL. The decay factor is not the most relevant parameter and can be easily taken into account analytically. The absence of absorption makes the relative enhancement of radiation into free space equivalent to Purcell's factor. This equivalence allows us to concentrate on the hyperlensing effect, i.e., on emission enhancement and matching of our structure to free space. Our model source is a very short dipole antenna of perfectly conducting wire with bulbs mimicking the Hertzian dipole at the simulation frequency.

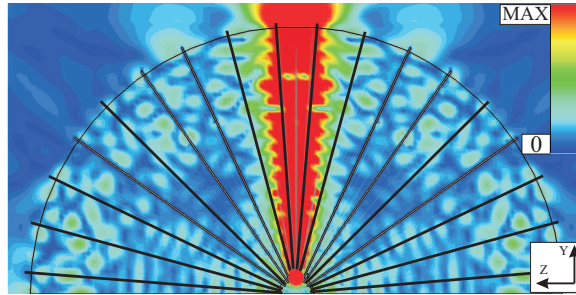


FIG. 2: Electric field amplitude distribution in the H -plane (vertical cross section orthogonal to the dipole) produced by a dipole located in between the internal ends of perfect nanowires of radius $r_0 = 25$ nm forming our HL. Its host material (between $r_1 = 0.5 \mu\text{m}$ and $R = 10 \mu\text{m}$) is glass.

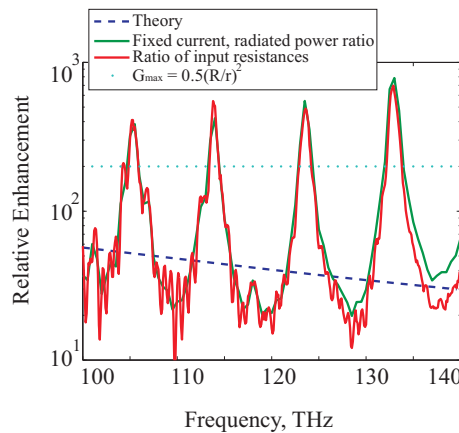


FIG. 3: Relative enhancement of radiation by a transverse dipole due to the presence of a HL of perfect wires calculated 1) directly via the radiated power spectrum (green curve) and 2) through the input resistance of a short wire dipole (red curve). The structure is the same as in Fig. 2. The theoretical blue dashed curve and the green dotted line are explained in the main text.

First, we calculate the field distributions to inspect if the wave beam divergence is sufficient to prevent strong reflections from the effective surface of the HL. For divergence angles within the range $\phi = 6\text{--}9^\circ$ the concept of HL turned out to be fully adequate, and the result weakly depends on ϕ over this interval of values. The result weakly depends on the exact location of the transverse dipole embedded into the WM up to 250 nm from the internal cavity r_1 . However, if the dipole is moved to the central cavity to the distance more than 250 nm, Purcell's factor drops to unity. Also, the radiation decreases if $\phi < 6^\circ$ i.e. when the HL approaches to a block of parallel nanowires. In Fig. 2 a color map illustrates the hyperlensing of the dipole radiation for optimal divergence angle $\phi = 10^\circ$. The horizontal dipole is located on top of the internal cavity r_1 in between two central nanowires and radiates at the frequency 120 THz which is between the bands of Fabry-Perot resonances. In both E - and H -planes we observed a sufficient width of the main radiation beam. The reflection from the effective surface of the HL in our simulations fits the estimation $t \approx t_{\text{HL}}$.

For a lossless HL G_{HL} can be calculated in two ways: via the input resistance of the antenna and via the far-zone radiated power. Both these values were calculated and normalized to the corresponding values simulated for the same dipole when the HL is absent. In the first case we keep the same input voltage of the antenna in the absence or presence of the HL. In the second case we fix the antenna current. The coincidence of two results is expected at low frequencies where the short wire antenna is close to the Hertzian dipole. This equivalence is seen in Fig. 3 at 100–130 THz where the red and green curves nearly coincide (besides the small ripples of the red curve which are numeric errors). In this plot we observe several Fabry-Perot resonances at which the HL gain reaches very high values. These values are, however, hardly relevant for the thermal radiation because the emitter mimicking the BB will absorb all incoming waves. Therefore, the Fabry-Perot resonances in the HL in a more realistic configuration will be greatly suppressed. The blue curve shows the theoretical estimation for the HL gain calculated in accordance to Eq. (3) with the factor $(2/3) \exp(-2\alpha R)$ excluded because only single orientation of the dipole in a lossless HL is considered in the simulations. In the range 100–130 THz our theoretical estimation agrees with the simulated gain when averaged between the Fabry-Perot resonances.

To conclude, we have suggested a structure that greatly enhances the radiative heat power produced by a small thermal emitter, which may go far beyond the limit enforced by Planck's law for the same radiator in free space. This is achieved by centering the emitter at the focal point of a hyperlens, which transforms emitter's near field into propagating waves which are matched well to free space and efficiently irradiated. However, the structure suggested in this paper still radiates less than a BB with the same size as that of the hyperlens and the same temperature as that of the emitter. A theoretical possibility to overcome this restriction for bodies of constrained radius is reserved for a future work (see [31]).

-
- [1] R. Carminati and J. J. Greffet, *Phys. Rev. Lett.* **82**, 1660 (1999).
 - [2] J. J. Greffet, R. Carminati, K. Joulain, J. P. Mulet, S. Mainguy, and Y. Chen, *Nature* **416**, 61 (2002).
 - [3] Y.-B. Chen and Z.M. Zhang, *Optics Comm.* **269**, 411 (2007).
 - [4] P. Ben-Abdallah, S.-A. Biehs, and K. Joulain, *Phys. Rev. Lett.* **107**, 114301 (2011).
 - [5] C. Fu and Zh. M. Zhang, *Front. Energy Power Eng. China* **3**, 11 (2009).
 - [6] Y. Guo, C. L. Cortes, S. Molesky, and Z. Jacob, *Appl. Phys. Lett.*, **101**, 131106 (2012).
 - [7] Y. Guo and Z. Jacob, *Optic Express* **21**, 15014 (2013).
 - [8] K. Joulain, J. Drevillon, and P. Ben-Abdallah, *Phys. Rev. B* **81**, 165119 (2010).
 - [9] S.-A. Biehs, M. Tschikin, R. Messina, and P. Ben-Abdallah, *Appl. Phys. Lett.* **102**, 131106 (2013).
 - [10] M. Florescu, K. Busch, and J. P. Dowling, *Phys. Rev. B* **75**, 201101(R) (2007).
 - [11] T. Trupke, P. Würfel, and M.A. Green, *Appl. Phys. Lett.* **84**, 1997 (2004).
 - [12] S. M. Rytov, *Theory of electric fluctuations and thermal radiation*, Electronics Research Directorate, Air Force Cambridge, Air Research and Development Command, USA (1959).
 - [13] Z. Yu, N. P. Sergeant, T. Skauli, G. Zhang, H. Wang, and S. Fan, *Nature Comm.* **4**, 1730 (2013).
 - [14] L. Novotny and B. Hecht, *Principle of Nanooptics*, Cambridge University Press, NY, USA (2006).
 - [15] E. Yablonovitch, *Phys. Rev. Lett.* **58**, 2059 (1987).
 - [16] Felsen, L. B. and N. Marcuvitz, *Radiation and Scattering of Waves*, Prentice-Hall, Englewood Cliffs, N. J. (1973).
 - [17] A. N. Poddubny, P. A. Belov, and Y. S. Kivshar, *Phys. Rev. B* **87**, 035136 (2013).
 - [18] I. S. Nefedov, C. A. Valagiannopoulos, S. M. Hashemi, and E. I. Nefedov, *Sc. Rep.* **3**, 2662 (2013).
 - [19] I. S. Nefedov, C. A. Valagiannopoulos, and L. Melnikov, *J. Opt.* **15**, 114003 (2013).
 - [20] I.S. Nefedov and L. A. Melnikov, arXiv:1402.3507 (2014).
 - [21] Z. Liu, H. Lee, Y. Xiong, C. Sun, and X. Zhang, *Science* **315**, 1686 (2007).
 - [22] E. E. Narimanov, V. Shalaev, *Nature* **447**, 266 (2007).
 - [23] J. Yao, Z. Liu, Y. Liu, Y. Wang, C. Sun, G. Bartal, A. M. Stacy, X. Zhang, *Science* **321**, 930 (2008).
 - [24] A. V. Kildishev and E. E. Narimanov, *Optic Letters* **32**, 3432 (2007).
 - [25] C. R. Simovski, P. A. Belov, A. V. Atraschenko, and Yu. S. Kivshar, *Advanced Materials* **24**, 4229 (2012).
 - [26] M.G. Silveirinha, *Phys. Rev. E* **73**, 046612 (2006).
 - [27] M. G. Silveirinha and S. I. Maslovski, *Phys. Rev. B* **85**, 155125 (2012).
 - [28] Y. Li, and M. G. Silveirinha, *IEEE Antennas And Wireless Propagation Letters* **12**, 401 (2013).
 - [29] S. I. Maslovski and M. G. Silveirinha, *Phys. Rev. B* **80**, 245101 (2009).
 - [30] *Handbook of Optical Constants of Solids*, E.D. Palik, Ed., Academic Press, San Diego, CA, USA (1998).
 - [31] S.I. Maslovski and C.R. Simovski, arXiv:1401.4013 (2014).